Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Thursday 10 November 2016 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

11 pages

8816 – 7301

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

Let $f(x) = x^2 - 4x + 5$.

(a) Find the equation of the axis of symmetry of the graph of f. [2]

The function can also be expressed in the form $f(x) = (x - h)^2 + k$.

(b) (i) Write down the value of h.

(ii) Find the value of k. [4]

2. [Maximum mark: 5]

Let $\sin\theta = \frac{\sqrt{5}}{3}$, where θ is acute.

(a) Find $\cos\theta$. [3]

(b) Find $\cos 2\theta$. [2]

3. [Maximum mark: 7]

The values in the fourth row of Pascal's triangle are shown in the following table.

1	4	6	4	1

(a) Write down the values in the fifth row of Pascal's triangle. [2]

(b) Hence or otherwise, find the term in x^3 in the expansion of $(2x + 3)^5$. [5]

4. [Maximum mark: 7]

The position vectors of points P and Q are $\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ and $7\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$ respectively.

(a) Find a vector equation of the line that passes through P and Q. [4]

(b) The line through P and Q is perpendicular to the vector $2\boldsymbol{i} + n\boldsymbol{k}$. Find the value of n. [3]

5. [Maximum mark: 6]

Events A and B are independent with $\mathrm{P}(A \cap B) = 0.2$ and $\mathrm{P}(A' \cap B) = 0.6$.

(a) Find $\mathrm{P}(B)$. [2]

(b) Find $\mathrm{P}(A \cup B)$. [4]

6. [Maximum mark: 7]

Let $f'(x) = \sin^3(2x)\cos(2x)$. Find $f(x)$, given that $f\left(\frac{\pi}{4}\right) = 1$.

7. [Maximum mark: 7]

Let $f(x) = m - \frac{1}{x}$, for $x \neq 0$. The line $y = x - m$ intersects the graph of f in two distinct points. Find the possible values of m.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Let $\vec{OA} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$ and $\vec{OB} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$.

(a) (i) Find $\vec{AB}$.

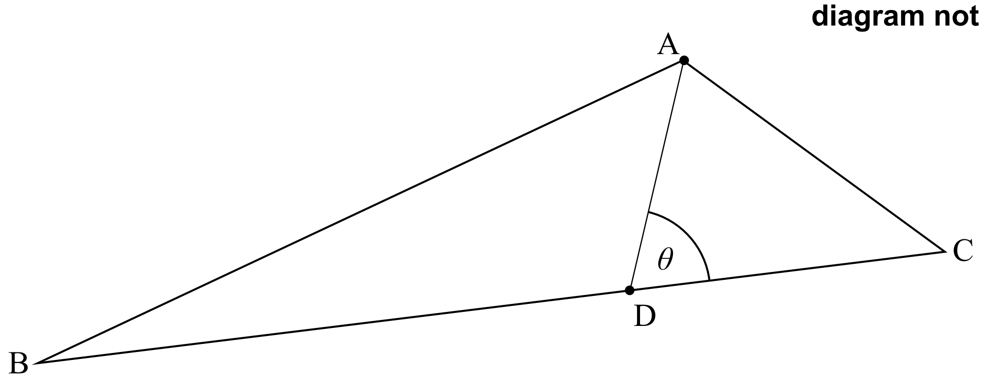
(ii) Find $\left|\vec{AB}\right|$. [4]

The point C is such that $\vec{AC} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$.

(b) Show that the coordinates of C are $(-2, 1, 3)$. [1]

The following diagram shows triangle ABC. Let D be a point on [BC], with acute angle $\text{ADC} = \theta$.

diagram not to scale



(c) Write down an expression in terms of θ for

(i) angle ADB;

(ii) area of triangle ABD. [2]

(d) Given that $\dfrac{\text{area } \Delta\text{ABD}}{\text{area } \Delta\text{ACD}} = 3$, show that $\dfrac{\text{BD}}{\text{BC}} = \dfrac{3}{4}$. [5]

(e) Hence or otherwise, find the coordinates of point D. [4]

Turn over

Do **not** write solutions on this page.

9. [Maximum mark: 13]

The first two terms of an infinite geometric sequence, in order, are

$$2\log_2 x,\ \log_2 x, \text{ where } x > 0.$$

(a) Find r. [2]

(b) Show that the sum of the infinite sequence is $4\log_2 x$. [2]

The first three terms of an arithmetic sequence, in order, are

$$\log_2 x,\ \log_2\left(\frac{x}{2}\right),\ \log_2\left(\frac{x}{4}\right), \text{ where } x > 0.$$

(c) Find d, giving your answer as an integer. [4]

Let S_{12} be the sum of the first 12 terms of the arithmetic sequence.

(d) Show that $S_{12} = 12\log_2 x - 66$. [2]

(e) Given that S_{12} is equal to half the sum of the infinite geometric sequence, find x, giving your answer in the form 2^p, where $p \in \mathbb{Q}$. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

Let $f(x) = \cos x$.

(a) (i) Find the first four derivatives of $f(x)$.

(ii) Find $f^{(19)}(x)$. [4]

Let $g(x) = x^k$, where $k \in \mathbb{Z}^+$.

(b) (i) Find the first three derivatives of $g(x)$.

(ii) Given that $g^{(19)}(x) = \frac{k!}{(k-p)!}\left(x^{k-19}\right)$, find p. [5]

Let $k = 21$ and $h(x) = \left(f^{(19)}(x) \times g^{(19)}(x)\right)$.

(c) (i) Find $h'(x)$.

(ii) Hence, show that $h'(\pi) = \frac{-21!}{2}\pi^2$. [7]